The Teleparallel version of Horndeski gravity

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Outline

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Galileons (in Minkowski background)

- Horndeski theory is the most general scalar tensor theory with one scalar field which leads to second order field eqs.
- The most straightforward way to obtain Horndeski theory is by considering Galileons; i.e. scalar fields that are invariant under the Galilean shift symmetry $\phi \to \phi + b_\mu x^\mu + c$.
- The most general Lagrangian that has the above property and gives second order field equations is

Lagrangian 2nd order Field eqs. for Minkowski

$$L = c_1\phi + c_2 X - c_3 X \Box\phi + c_4 X\left[(\Box\phi)^2 - \partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi\right] - c_5 X\left[(\Box\phi)^3 - 3(\Box\phi)\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi + 2\partial_\mu\partial_\nu\phi\partial^\nu\partial^\lambda\phi\partial_\lambda\partial^\mu\phi\right],$$

where $X = -1/2\partial^\mu\phi\partial_\mu\phi$ is the kinetic term.

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Covariant Galileons

- To introduce gravity we promote $\eta_{\mu\nu} \to g_{\mu\nu}$ and $\partial_\mu \to \mathring{\nabla}_\mu$.
- Doing that, one needs to be careful since covariant derivatives do not commute. The naive covariantisation leads to higher derivatives in the field equations.
- Then, we need to add some correction terms, which leads the following Lagrangian

$$\mathring{L} = c_1\phi + c_2 X - c_3 X \mathring{\Box}\phi + \frac{c_4}{2}X^2\mathring{R} + c_4 X\left[(\mathring{\Box})^2 - \phi^{\mu\nu}\phi_{\mu\nu}\right] + c_5 X^2 \mathring{G}^{\mu\nu}\phi_{\mu\nu} - \frac{c_5}{3}X\left[(\mathring{\Box})^3 - 3\mathring{\Box}\phi^{\mu\nu}\phi_{\mu\nu} + 2\phi_{\mu\nu}\phi^{\nu\lambda}\phi^{\mu}_{\lambda}\right].$$

and $\phi_{\mu\nu} = \mathring{\nabla}_\mu\mathring{\nabla}_\nu\phi$. Over-circles mean that it's computed with respect to Levi Civita connection.

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Horndeski gravity

- A generalized version of the above is the Horndeski one

$$\mathring{L}_2 = G_2(\phi, X)\,, \quad \mathring{L}_3 = -G_3(\phi, X)\,(\mathring{\Box}\phi)\,\phi\,,$$
$$\mathring{L}_4 = G_4(\phi, X)\mathring{R} + G_{4,X}(\phi, X)\left[(\mathring{\Box}\phi)^2 - \mathring{\nabla}_\mu\mathring{\nabla}_\nu\phi\mathring{\nabla}^\mu\mathring{\nabla}^\nu\phi\right]\,,$$
$$\mathring{L}_5 = G_5(\phi, X)\mathring{G}_{\mu\nu}\mathring{\nabla}^\mu\mathring{\nabla}^\nu\phi - \frac{1}{6}G_{5,X}(\phi, X)\Big[(\mathring{\Box}\phi)^3 + 2\mathring{\nabla}_\nu\mathring{\nabla}_\mu\phi\mathring{\nabla}^\nu\mathring{\nabla}^\lambda\phi\mathring{\nabla}_\lambda\mathring{\nabla}^\mu\phi - 3\mathring{\Box}\phi\mathring{\nabla}_\mu\mathring{\nabla}_\nu\phi\mathring{\nabla}^\mu\mathring{\nabla}^\nu\phi\Big]\,,$$

where the total Lagrangian is $\mathring{L} = \sum_{i=2}^{5}\mathring{L}_i$.

Gravitational waves in standard Horndeski

- The speed of propagation of gravitational waves for Horndeski gravity in flat FLRW background is

Speed of GW in standard Horndeski

$$c_T^2 = \frac{G_4 - X(\ddot{\phi} G_{5,X} + G_{5,\phi})}{G_4 - 2XG_{4,X} - X(H\dot{\phi}G_{5,X} - G_{5,\phi})}$$

- According to GW observations *Prog.Theor.Phys. 126 (2011), 511-529*, it is required that $c_T = 1$ which is achieved only if $G_4(\phi, X) = G_4(\phi)$ and $G_5 = \text{constant}$

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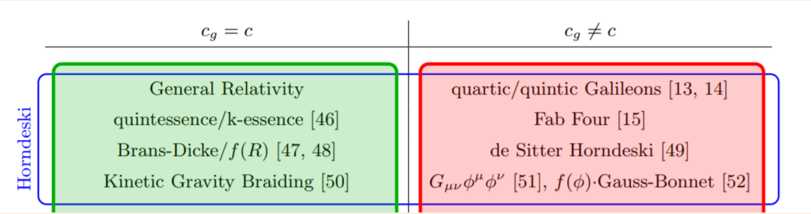

Figure: Summary of the viable (left) and non-viable (right) scalar-tensor theories after GW170817. PRL 119, 251304 (2017)

Outline

Teleparallel equivalent of GR

- **Teleparallel equivalent of GR** (TEGR) is an alternative and equivalent formulation of gravity from GR which uses the **tetrad** formalism and its connection has **zero curvature** and **non-zero torsion**.
- **Tetrads** (or vierbein) $e^a{}_\mu$ are the linear basis on the spacetime manifold, and at each point of the spacetime, tetrads gives us basis for vectors on the tangent space.
- Tetrads satisfy the **orthogonality condition**; $E_m{}^\mu e^n{}_\mu = \delta^n_m$ and $E_m{}^\nu e^m{}_\mu = \delta^\nu_\mu$ and the metric can be reconstructed via

$$g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu \,,$$

where η_{ab} is the Minkowski metric.

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Torsion tensor

- By using the Teleparallel connection (Weitzenböck connection), one can express the torsion tensor as follows

Torsion tensor

$$T^{\rho}{}_{\mu\nu} = \tilde{\Gamma}^{\rho}{}_{\nu\mu} - \tilde{\Gamma}^{\rho}{}_{\mu\nu} = E_A{}^{\rho}\left(e^A{}_{\nu,\mu} - e^A{}_{\mu,\nu} + \omega^A{}_{B\mu}e^B{}_{\nu} - \omega^A{}_{B\nu}e^B{}_{\mu}\right).$$

- The torsion tensor is generally non-vanishing, and transforms covariantly under both diffeomorphisms and local Lorentz transformations.
- The pure tetrad formalism was the initial framework used for TG, which chooses a specific frame where the spin connection $\omega^a{}_{b\mu}$ vanishes. Be careful choosing the correct tetrad which is compatible with this gauge.

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Ricci theorem in Teleparallel gravity

- By splitting the curvature tensor and contracting it with the metric $g^{\mu\nu}R^{\lambda}{}_{\mu\lambda\nu} \equiv R$ (Ricci scalar-general one), one gets

Ricci theorem final in TEGR

$$R = \mathring{R} + T - B = 0 \rightarrow \mathring{R} = -T + B\,,$$

where $B = \frac{2}{e}\partial_{\mu}(eT^{\mu})$ is a boundary term in the action (see later) and $T = \frac{1}{4}T^{\rho}{}_{\mu\nu}T_{\rho}{}^{\mu\nu} + \frac{1}{2}T^{\rho}{}_{\mu\nu}T^{\nu\mu}{}_{\rho} - T^{\lambda}{}_{\lambda\mu}T_{\nu}{}^{\nu\mu}$ is the **scalar torsion.**

Important remark here

The Ricci scalar computed from the Levi-Civita connection $\mathring{R}$ differs from the scalar torsion T by a boundary term B.

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Teleparallel equivalent of General Relativity action

- The TEGR action is formulated based on the torsion scalar T, namely

$$S_{\mathrm{TEGR}} = \int \left[T + 2\kappa^2 L_{\mathrm{m}}\right] e\, d^4x\,.$$

where $\kappa^2 = 8\pi G$, $e = \det(e^a_\mu) = \sqrt{-g}$ and L_{m} is any matter Lagrangian.

- T and the scalar curvature $\overset{\circ}{R}$ differs by a boundary term B as $\overset{\circ}{R} = -T + B$ so:

Equivalence between field equations

The teleparallel field equations are equivalent to the Einstein field equations.

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Two different ways of understanding gravity

Equivalence on their field equations

VERY IMPORTANT POINT: TEGR has the same equations as GR, so **CLASSICALLY** it is impossible to make any observation to distinguish between them.

Validity of TEGR

VERY IMPORTANT POINT: All classical experiments already done, that have confirmed GR, also can be understood as a confirmation of TEGR.

What happens if we modify TEGR?

If we modify the TEGR action, a priori there is no equivalence between modified theories from GR and modified Teleparallel theories.

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Torsion decomposition

The torsion tensor can be decomposed in its irreducible parts as

$$a_\mu = \frac{1}{6}\epsilon_{\mu\nu\sigma\rho}T^{\nu\sigma\rho}\,, \quad v_\mu = T^\sigma{}_{\sigma\mu}\,,$$
$$t_{\sigma\mu\nu} = \frac{1}{2}\left(T_{\sigma\mu\nu} + T_{\mu\sigma\nu}\right) + \frac{1}{6}\left(g_{\nu\sigma}v_\mu + g_{\nu\mu}v_\sigma\right) - \frac{1}{3}g_{\sigma\mu}v_\nu\,,$$

where $\epsilon_{\mu\nu\sigma\rho}$ is the totally anti-symmetric Levi-Civita symbol. From these we build the scalars

$$T_{\text{ax}} = a_\mu a^\mu\,, \quad T_{\text{vec}} = v_\mu v^\mu\,, \quad T_{\text{ten}} = t_{\sigma\mu\nu}t^{\sigma\mu\nu}\,,$$

and the torsion scalar is a linear combination

$$T = \frac{3}{2}T_{\text{ax}} + \frac{2}{3}T_{\text{ten}} - \frac{2}{3}T_{\text{vec}}\,.$$

Conditions for the theory

Condition 1

The resulting field equations must be at most second order in terms of derivatives of the tetrad fields (or equivalently second order in terms of metric tensor derivatives).

Condition 2

The scalar invariants should not be parity violating.

Condition 3

The field equations must be covariant under local Lorentz transformations.

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Contractions of the torsion tensor can at most be quadratic.

Any number of contractions of the irreducible parts of the torsion tensor will result in second order field equations. This means that an infinite number of terms can be formed in Teleparallel gravity that give rise to second order field equations. However, it is unclear how physical such higher order contributions will be.

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Covariantisation procedure

GR	Teleparallel
$\eta_{\mu\nu} \to g_{\mu\nu}$	$e^a{}_\mu \to h^a{}_\mu$
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Table: Covariantisation prescription

Following the same procedure as before, we start from

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Teleparallel Horndeski construction

- Since $\mathcal{D}_\mu$ coincides with $\mathring{\nabla}_\mu$ computed with the Levi-Civita connection, then the Teleparallel Lagrangians $\sum_{i=3}^{5}\mathcal{L}$ are identical to $\sum_{i=3}^{5}\mathring{L}_i$.
- However, when one is considering Teleparallel gravity, $\mathcal{L}_2$ would be different to $\mathring{L}_2$ since there are more scalars that one can construct which satisfies the conditions. Notation: $\mathcal{L}_2 = \mathring{L}_2 = G_2(\phi, X)$ and extra term $\mathcal{L}_{\rm Tele}$

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Teleparallel Horndeski construction - Constructing $\mathcal{L}_{\rm Tele}$

- Taking quadratic contractions of the torsion tensor, the most general Lagrangian of Teleparallel gravity satisfying the conditions turns out to be $f(T_{\text{ax}}, T_{\text{vec}}, T_{\text{ten}})$ (without a scalar field)[1].
- If one adds the scalar field, one can construct the following **7 extra independent scalars**:

Possible independent scalars

$$I_2 = v^{\mu}\phi_{;\mu}\,,\quad J_1 = a^{\mu}a^{\nu}\phi_{;\mu}\phi_{;\nu}\,,\quad J_3 = v_{\sigma}t^{\sigma\mu\nu}\phi_{;\mu}\phi_{;\nu}\,,$$

$$J_5 = t^{\sigma\mu\nu}t_{\sigma}{}^{\bar{\mu}}{}_{\nu}\phi_{;\mu}\phi_{;\bar{\mu}}\,,\quad J_6 = t^{\sigma\mu\nu}t_{\sigma}{}^{\bar{\mu}\bar{\nu}}\phi_{;\mu}\phi_{;\nu}\phi_{;\bar{\mu}}\phi_{;\bar{\nu}}\,,$$

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- Then, the extra term $\mathcal{L}_{\rm Tele}$ related to Teleparallel gravity will be equal to

Extra term in $\mathcal{L}_2$ in Teleparallel Horndeski

$$\mathcal{L}_{\text{Tele}} = G_{\text{Tele}}(\phi, X, T, T_{\text{ax}}, T_{\text{vec}}, I_2, J_1, J_3, J_5, J_6, J_8, J_{10})\,.$$

- It is equivalent to consider $T_{\rm ten}$ to T above.
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Final Lagrangian Teleparallel Horndeski

The final Lagrangian reads[2]

Teleparallel Horndeski Lagrangian

$$\mathcal{L} = \sum_{i=2}^{5} \mathcal{L}_i + \mathcal{L}_{\text{Tele}} = \text{Horndeski} + \mathcal{L}_{\text{Tele}}\,,$$

where

$$\begin{aligned}
\mathcal{L}_{\text{Tele}} &= G_{\text{Tele}}(\phi, X, T, T_{\text{ax}}, T_{\text{vec}}, I_2, J_1, J_3, J_5, J_6, J_8, J_{10})\,,\\
\mathcal{L}_2 &= G_2(\phi, X)\,, \quad \mathcal{L}_3 = G_3(\phi, X)\mathring{\Box}\phi\,,\\
\mathcal{L}_4 &= G_4(\phi, X)\,(-T + B) + G_{4,X}(\phi, X)\left[(\mathring{\Box}\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right]\,,\\
\mathcal{L}_5 &= G_5(\phi, X)\mathcal{G}_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}G_{5,X}(\phi, X)\Big[(\mathring{\Box}\phi)^3 + 2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\alpha}\phi_{;\alpha}{}^{\mu}\\
&\quad - 3\phi_{;\mu\nu}\phi^{\mu\nu}\,(\mathring{\Box}\phi)\Big]\,.
\end{aligned}$$

[2] S. Bahamonde, K. F. Dialektopoulos and J. Levi Said, Phys. Rev. D **100** (2019)

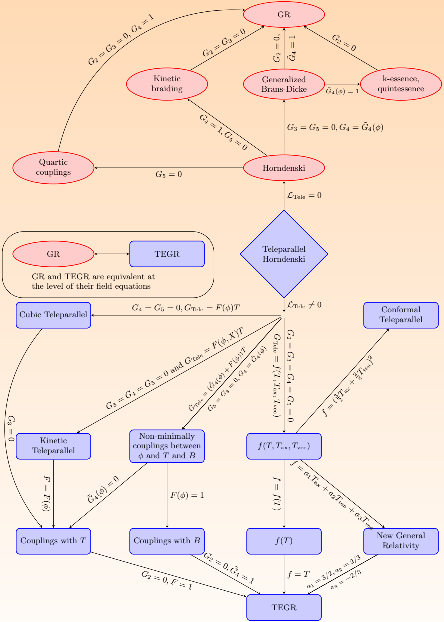

FIG. 1: Relationship between Teleparallel Horndenski and various theories.

Gravitational waves in Teleparallel Horndeski in flat FLRW background

- By considering tensorial perturbations only ($\delta e^k{}_\mu = \frac{1}{2} a\, \delta^i_\mu \delta^{kj} h_{ij}$) and after some cumbersome calculations, one gets the following wave equation

$$\ddot{h}_{ij} + (3 + \alpha_M)\, H \dot{h}_{ij} - (1 + \alpha_T)\, \frac{k^2}{a^2} h_{ij} = 0\,,$$

where $\alpha_T = c_T^2 - 1$ and the speed of GW being equal to[3]

Speed of GW in Teleparallel Horndeski

$$\alpha_T = \frac{2X}{M_*^2}\left(2G_{4,X} - 2G_{5,\phi} - G_{5,X}(\ddot{\phi} - \dot{\phi} H) - 2G_{\mathrm{Tele},J_8} - \frac{1}{2} G_{\mathrm{Tele},J_5}\right).$$

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Reviving Horndeski using Teleparallel gravity

- As we said before, for $G_{\rm Tele}=0$ (standard case), one gets that to achieve a theory consistent with the GW observations $c_T=1$, one requires $G_5(\phi,X)=\text{constant}$ and $G_4(\phi,X)=G_4(\phi)$. Hence, Horndeski gravity is highly constraint.
- If one has Teleparallel Horndeski, c_T^2 is corrected and then when does no need those conditions. Indeed, $G_5=G_5(\phi)$ and $G_4=G_4(\phi,X)$ still respect this condition:

Teleparallel Lagrangian respecting $c_T=1$ ($\alpha_T=0$)

$$\begin{aligned}\mathcal{L} = {} & \tilde{G}_{\rm tele}(\phi,X,T,T_{\rm vec},T_{\rm ax},I_2,J_1,J_3,J_6,J_8-4J_5,J_{10}) + G_2(\phi,X) + G_3(\phi,X)\Box\phi\,,\\ & + G_4(\phi,X)\left(-T+B\right) + G_{4,X}\left[(\Box\phi)^2-\phi_{;\mu\nu}\phi^{;\mu\nu}+4J_5\right]\\ & + G_5(\phi)\mathcal{G}_{\mu\nu}\phi^{;\mu\nu} - 4J_5G_{5,\phi}\,.\end{aligned}$$

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Parametrized post-Newtonian limit

- Tool for testing the viability of metric theories of gravity, by a set of ten (usually constant) parameters.
- To do this here, one needs to adapt this method for tetrads[4]:

$$\overset{2}{e}_{00} = U\,,$$

$$\overset{2}{e}_{(ij)} = \gamma U \delta_{ij}\,,$$

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$$\overset{4}{e}_{00} = \frac{1}{2}(1-2\beta)U^2 + \frac{1}{2}(2+2\gamma+\alpha_3+\zeta_1-2\xi)\Phi_1 + (1+3\gamma-2\beta+\zeta_2+\xi)\Phi_2 + (1+\zeta_3)\Phi_3 + (3\gamma+3\zeta_4-2\xi)\Phi_4 - \xi\Phi_W - \frac{1}{2}(\zeta_1-2\xi)\mathfrak{A}\,.$$

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PPN - result

- The final result for our theory is[5]

$$\xi = \alpha_1 = \alpha_2 = \alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0\,,$$

- This indicates that our theory is fully conservative, i.e., it does not exhibit any preferred-frame or preferred-location effects, or a violation of the conservation of total energy-momentum.
- The only PPN parameters, for which we obtain a deviation from their general relativity values are α and β (see our paper for more details). One can find theories being identical to GR at PPN: $\alpha = \beta = 1$

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Outline

Conclusions

- Horndeski is the most general second order field equations with one scalar field. This theory was highly constraint after GW170817.
- We formulate an analogue version of it in the Teleparallel framework. Since torsion has first derivatives of tetrads, there are more terms that respects the second order condition.
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